

MATHEMATICS

DIFFERENTIATION

1. Find $\frac{dy}{dx}$ for the following
 - (a) $y = x^3 + 2x + 3$
 - (b) $y = kx^2 + c$ where k & c are any two real constant
 - (c) $y = x^2 + \sin x$
 - (d) $y = x + \frac{1}{x} + \ln x + 3^x$
 - (e) $y = 2 \cos x + 3 \sin x + \tan x$
 - (f) $y = tx^2 + 1$ where t is independent on x
 - (g) $y = xt + 2t + x$, find $\frac{dy}{dt}$
 - (h) $y = tx^2 + 1$ if t is dependent on x such that $t = (2x - 1)$
 - (i) $y = \sec x + \tan x + x^{\frac{1}{3}} + \frac{1}{x} + x^{\frac{1}{2}}$

2. Find $\frac{dy}{dx}$ for the following
 - (a) $y = xe^x$
 - (b) $y = x^2 \sin x + x \ln x$
 - (c) $y = x^n \cos x + e^x \sin x$
 - (d) $y = t \sin x$ if t is independent on x
 - (e) $y = t \sin x$ if $t = xe^x$

3. Find $\frac{dy}{dx}$ for the following
 - (a) $y = \frac{x}{1+x}$
 - (b) $y = \frac{e^x}{1+x}$
 - (c) $y = \frac{e^x + 1}{1 + \sin x}$
 - (d) $y = \frac{t}{t+x}$ if t is independent on x
 - (e) $y = \frac{t}{t+x}$ if $t = e^x$

4. Find $\frac{dy}{dx}$ if
 - (a) $y = \sin(x + y)$
 - (b) $y = \tan(e^{(x+y)})$

5. If $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$, Find $\frac{dy}{dx}$, Ans. $\tan \theta / 2$

6. If $x = a \cos^3 \theta$, $y = a \sin^3 \theta$, Find $\frac{dy}{dx}$ Ans. $-\tan \theta$

7. If $x = \log t + \sin t$, $y = e^t + \cos t$, find $\frac{dy}{dx}$, Ans. $\frac{t(e^t - \sin t)}{1 + t \cos t}$

8. If $y = x^x$, Find $\frac{dy}{dx}$ Ans. $x^x(1 + \log x)$

9. If $y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$,
 Prove that $\frac{dy}{dx} = (\sin x)^{\cos x} \{ \cot x \cdot \cos x - \log(\sin x)^{\sin x} \} + (\cos x)^{\sin x} \{ \log(\cos x)^{\cos x} - \tan x \cdot \sin x \}$

10. If $y = \cos \sqrt{\sin \sqrt{x}}$, Find $\frac{dy}{dx}$
11. If $xy = x^3 + y^3$, find $\frac{dy}{dx}$
12. If $x + y = \sin(xy)$, Find $\frac{dy}{dx}$
13. If $y = \tan(x + y)$, Find $\frac{dy}{dx}$

14. If $x^3 + y^3 = \sin(x + y)$, Find $\frac{dy}{dx}$
15. If $x = y \log(xy)$, Find $\frac{dy}{dx}$
16. If $\sin y = x \sin(a + y)$, Prove That $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$
17. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \infty}}}$. Prove that $\frac{dy}{dx} = \frac{\cos x}{2y - 1}$
18. If $x = y + \frac{1}{y + \frac{1}{y + \frac{1}{\dots \infty}}}$, Prove that $\frac{dy}{dx} = 2x^2 + y^2 - 3xy$
19. If $\cos y = x \cos(b + y)$, prove that $\frac{dy}{dx} = \frac{\cos^2(b + y)}{\sin b}$
20. If $y = \frac{e^x}{\log x}$, Find $\frac{dy}{dx}$
21. If $y = \sin(\cot x)$, Find $\frac{dy}{dx}$
22. If $y = \sin(\sqrt{\cos x})$, Find $\frac{dy}{dx}$
23. If $y = \sqrt{\sin \sqrt{x}}$, Find $\frac{dy}{dx}$

SOME MORE PROBLEMS

1. Find $\frac{dy}{dx}$, from the first principle, where y is defined by
 (i) $y = x^{-3/4}$ (ii) $y = (a + bx)^{-1/3}$
2. Using the first principle, find the derivative with respect to x, of
 (i) $\tan 2x$ (ii) $\tan \sqrt{x}$ (iii) $\cos \sqrt{x}$.
3. Find the derivative with respect to x of (using the first principle)
 (i) $\cos^{-1} x^2$ (ii) $\sin(x^2 + 1)$
4. Find $\frac{dy}{dx}$, where y is defined by
 (i) $y = \frac{5x}{\sqrt{1-x^2}} + \cos^2(2x + 1)$ (ii) $y = \sqrt{\frac{1+e^x}{1-e^x}}$
 (iii) $y = \sin^{-1} \sqrt{\frac{1+x^2}{2}}$ (iv) $y = \tan^{-1} \left[\frac{\sqrt{1+a^2x^2} - 1}{ax} \right]$.
5. Differentiate
 (i) $\sin^{-1} \left[x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2} \right]$ (ii) $\tan^{-1} \left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right]$ with respect to x.
6. Find the derivatives of
 (i) $e^x \log(1 + x^2)$ (ii) $\frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$
 (iii) $\tan^{-1} \sqrt{\frac{1+\sin x}{1-\sin x}}$ with respect to x.
7. Find $\frac{dy}{dx}$, when $x = \log(1 + t^2)$, $y = \tan^{-1} t$.
8. Find $\frac{dy}{dx}$ when $x^y = e^{x+y}$.

ANSWERS TO SOME MORE PROBLEMS

1. (i) $-\frac{3}{4}x^{-7/4}$ (ii) $-\frac{b}{3}(a+bx)^{-4/3}$
2. (i) $2 \sec^2 2x$ (ii) $\frac{1}{2\sqrt{x}} \sec^2 \sqrt{x}$ (iii) $-\frac{1}{2\sqrt{2}} \sin \sqrt{x}$
3. (i) $\frac{-2x}{\sqrt{1-x^4}}$ (ii) $2x \cos(x^2+1)$
4. (i) $\frac{5}{(1-x^2)^{3/2}} - 2 \sin(4x+2)$ (ii) $\frac{e^x}{(1+e^x)^{1/2}(1-e^x)^{3/2}}$
(iii) $\frac{x}{\sqrt{1-x^4}}$ (iv) $\frac{a}{2(1+a^2x^2)}$
5. (i) $\frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{x}\sqrt{1-x}}$ (ii) $\frac{-x}{\sqrt{1-x^4}}$
6. (i) $e^x \left[\log(1+x^2) + \frac{2x}{1+x^2} \right]$ (ii) $\frac{-8e^{4x}}{(e^{4x}-1)^2}$ (iii) $\frac{1}{2}$
7. $\frac{1}{2t}$ 8. $\frac{x-y}{x(\log x-1)}$ 9. (i) $\frac{1}{2y-1}$ (ii)
10. $\frac{y^2}{x(1-y \log x)}$ 10. $\frac{2}{3}$
11. (i) $e^{ax} \left[\frac{2a}{x} - \frac{1}{x^2} + a^2 \log x \right]$ (ii) $\frac{x}{(1-x)^{3/2}}$ (iii) 0
13. (i) $\frac{3}{\sqrt{1-x^2}}$ (ii) $\frac{x \cot\left(\frac{x^2}{3}+1\right)}{3\sqrt{\log\left(\frac{x^2}{3}-1\right)}}$
14. (i) $\frac{[\sin(x+y) + x \cos(x+y)]}{[\cos y - x \cos(x+y)]}$ (ii) $\frac{\left[y^{\cot x} \operatorname{cosec}^2 \log y - \frac{(\tan^{-1} x)^y y}{(1+x^2)\tan^{-1} x} \right]}{\left[y^{\cot x-1} \cot x + (\tan^{-1} x)^y \log \tan^{-1}(x) \right]}$